Notes on 2.6 (zeros of polynomials and Muller's method)

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Fundamental Theorem of Algebra
A degree $n \geq 1$ polynomial has exactly n roots (possibly complex, possibly with multiplicity greater than 1)

With this we can uniquely factor a polynomial

$$P(x) = a_n \prod_{i=1}^{k} (x - x_i)^{m_i}$$

$x_1, \ldots, x_k$ are the roots, $\sum_{i=1}^{k} m_i = n$ is the order of the polynomial, m_i is the multiplicity of the ith root

Horner's Method
To evaluate a polynomial at a point more efficiently than just plugging it in (kinda lame)

To compute $P(x_0)$ with $P = a_n x^n + \cdots a_0$ then let
$b_n = a_n$
$b_{n-1} = a_{n-1} + b_n x_0$
$b_{n-2} = a_{n-2} + b_{n-1} x_0$
\cdots

Then $b_0 = P(x_0)$ is what we want!

Proof
If Q is the polynomial from the b_i: $Q(x) = b_n x^{n-1} + \cdots + b_0$ then by multiplication: $(x - x_0)Q(x) = P(x) - b_0$

Therefore pluggin in $x = x_0$, we get $P(x_0) - b_0 = 0$

this leads to something called **Synthetic division** (which is kinda neat)

Muller's Method
Root finding method. Basically just secant method but uses parabolas instead.

Start with three initial guesses, the four point is by finding the intersection of the parabola with the x-axis (at the nearest point)